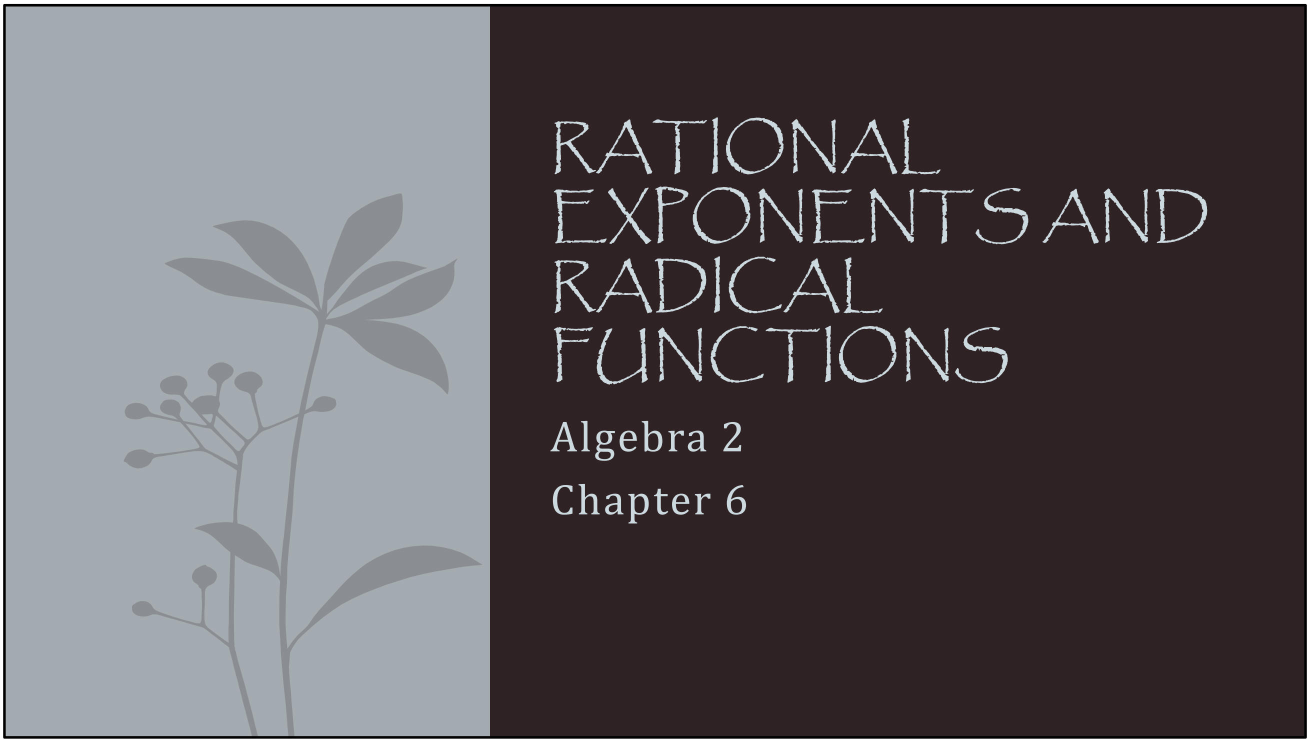
RATIONAL EXPONENTS AND RADICAL FUNCTIONS
Algebra 2
Chapter 6

Algebra II 6

- This Slideshow was developed to accompany the textbook
  - *Larson Algebra 2*
  - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
  - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

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6.1 Evaluate n^{th} Roots and Use Rational Exponents

- Root
  - If $a^2 = b$, then a is a square (2^{nd}) root of b
  - If $a^n = b$, then a is the n^{th} root of b
- Parts of a radical

Index → $\sqrt[3]{64}$ ← Radical Sign

Radicand

6.1 Evaluate n^{th} Roots and Use Rational Exponents

- Rational Exponents
  - $b^{1/n} = \sqrt[n]{b}$
  - $b^{m/n} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$

- Evaluate
  - $36^{1/2}$

$\sqrt{36} = 6$

6.1 Evaluate n^{th} Roots and Use Rational Exponents

- $\left(\frac{1}{8}\right)^{-\frac{1}{3}}$

- $27^{\frac{4}{3}}$

$\sqrt[3]{(1/8)^{-1}} = \sqrt[3]{8} = 2$
$\sqrt[3]{27^4} = 3^4 = 81$

6.1 Evaluate n^{th} Roots and Use Rational Exponents

- To find the roots by hand
  - Find the prime factorization of the radicand
  - Group the prime factors into groups of the same factor. Each group should have as many factors as the index.
  - For each complete group, you can move that factor out of the radical once (the group becomes one number)
  - If the index is even and the radicand is negative, the roots are not real

6.1 Evaluate n^{th} Roots and Use Rational Exponents

- $\sqrt[3]{64}$
- $\sqrt{36x^4}$

$\sqrt[3]{64} = \sqrt[3]{4*4*4} = 4$

$\sqrt{36x^4} = \sqrt{2*2*3*3*x*x*x*x} = \sqrt{(2*2)*(3*3)*(x*x)*(x*x)} = 2*3*x*x = 6x^2$

6.1 Evaluate n^{th} Roots and Use Rational Exponents

- Find roots with a calculator
  - The $\sqrt{x}$ or $\sqrt{}$ key is for square roots (either radicand then key or key then radicand depending on calculator)
  - The $\sqrt[x]{y}$ or $\sqrt[y]{x}$ or $\sqrt[x]{}$ is for any root (index → key → radicand OR radicand → key → index)
  - Try it with $\sqrt[4]{100}$

3.16

Quiz

- 6.1 Homework Quiz

6.2 Apply Properties of Rational Exponents

- Using Properties of Rational Exponents
  - $x^m \cdot x^n = x^{m+n}$
  - $(xy)^m = x^m y^m$
  - $(x^m)^n = x^{mn}$
  - $\frac{x^m}{x^n} = x^{m-n}$
  - $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$
  - $x^{-m} = \frac{1}{x^m}$

6.2 Apply Properties of Rational Exponents

- $6^{1/2} \cdot 6^{1/3}$
- $(27^{1/3} \cdot 6^{1/4})^2$

→ $6^{1/2 + 1/3}$ → $6^{3/6 + 2/6}$ → $6^{5/6}$

→ $(27^{1/3})^2 \cdot (6^{1/4})^2$ → $27^{2/3} \cdot 6^{1/2}$ → $9 \cdot 6^{1/2}$

6.2 Apply Properties of Rational Exponents

- $(4^3 \cdot w^3)^{-1/3}$
- $\frac{t}{t^{\frac{3}{4}}}$

→ $4^{3 \cdot -1/3} \cdot w^{3 \cdot -1/3}$ → $4^{-1} \cdot w^{-1}$ → ¼ · 1/w → 1/(4w)
→ $t^{1-3/4}$ → $t^{1/4}$

6.2 Apply Properties of Rational Exponents

- Using Properties of Radicals
  - Product Property →
    - $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
  - Quotient Property →
    - $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

6.2 Apply Properties of Rational Exponents

- $\sqrt[3]{25} \cdot \sqrt[3]{5}$
- $\dfrac{\sqrt[3]{32x}}{\sqrt[3]{4x}}$

$\sqrt[3]{25*5} = \sqrt[3]{125} = 5$

$\sqrt[3]{32x/4x} = \sqrt[3]{8} = 2$

6.2 Apply Properties of Rational Exponents

- Writing Radicals in Simplest Form
  - Remove any perfect roots
  - Rationalize denominators
- $\sqrt[4]{64}$
- $(16g^4h^2)^{1/2}$

$\sqrt[4]{2*2*2*2*2*2} = \sqrt[4]{(2*2*2*2)*2*2} = 2\sqrt[4]{4}$

6.2 Apply Properties of Rational Exponents

- $\sqrt[4]{\frac{7}{8}}$

$\sqrt[4]{7}/\sqrt[4]{8} = 7^{1/4}/8^{1/4} = 7^{1/4}*8^{3/4}/8^{1/4}*8^{3/4} = \sqrt[4]{(7*8^3)}/8 = \sqrt[4]{((2*2*2*2)*(2*2*2*2)*2*7)}/8 = 2*2*\sqrt[4]{(2*7)}/8 = \sqrt[4]{14}/2$

6.2 Apply Properties of Rational Exponents

- $\sqrt[5]{\frac{x^5}{y^8}}$
- $\frac{18rs^{\frac{2}{3}}}{6r^{\frac{1}{4}}t^{-3}}$

$3r^{1-1/4}s^{2/3}t^{0--3} = 3r^{3/4}s^{2/3}t^{3}$

6.2 Apply Properties of Rational Exponents

- Adding and Subtracting Roots and Radicals
  - Simplify the radicals
  - Combine like terms
- $5(4^{3/4}) - 3(4^{3/4})$

$\rightarrow 2(4^{3/4})$

6.2 Apply Properties of Rational Exponents

- $\sqrt[3]{81} - \sqrt[3]{3}$
- $2\sqrt[4]{6x^5} + x\sqrt[4]{6x}$

$\sqrt[3]{3*3*3*3} - \sqrt[3]{3} = 3\sqrt[3]{3} - \sqrt[3]{3} = 2\sqrt[3]{3}$

Quiz

- 6.2 Homework Quiz

6.3 Perform Function Operations and Composition

- Sometimes for your problems you need to repeat several calculations over and over again (think science class).

- It would be quicker to combine all the equations that you are using into one equation first, so that you only have to do one equation each time instead of many.

6.3 Perform Function Operations and Composition

- Ways to combine functions
- Addition: $(f + g)(x) = f(x) + g(x)$
- Subtraction: $(f - g)(x) = f(x) - g(x)$
- Multiplication: $(f \cdot g)(x) = f(x) \cdot g(x)$
- Division: $(f/g)(x) = f(x) / g(x)$

6.3 Perform Function Operations and Composition

- Given $f(x) = 2x^{1/2} - 1$ and $g(x) = x^{1/2}$ find
  - $(f + g)(x)$
  - $(f - g)(x)$
  - $(f \cdot g)(x)$
  - $(f / g)(x)$

$3x^{1/2} - 1$ D: $x \geq 0$
$x^{1/2} - 1$ D: $x \geq 0$
$2x - x^{1/2}$ D: $x \geq 0$
$(2x^{1/2} - 1)/x^{1/2}$ D: $x > 0$

6.3 Perform Function Operations and Composition

- Composition
  - Put one function into the other. (Like substitution)
  - Written f(g(x))
  - Said "f of g of x"
  - Means that the output (range) of g is the input (domain) of f. Work from the inside out. Do g(x) first then f(x).
  - g(x) gets substituted into f(x)

6.3 Perform Function Operations and Composition

- Find $f(g(x))$ when $f(x) = 2x + 3$ and $g(x) = x^2$
- Find $g(g(x))$

Where the x is in f put the g function $f(g(x)) = 2(x^2) + 3$
Simplify $f(g(x)) = 2x^2 + 3$
Example: Find $g(f(x))$
$g(f(x)) = g(2x + 3) = (2x + 3)^2 = 4x^2 + 12x + 9$

Quiz

- 6.3 Homework Quiz

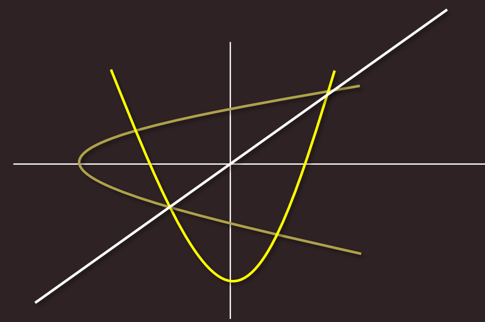
6.4 Use Inverse Functions

- Sometimes you want to do the opposite operation that a given function or equation gives you.

- To do the opposite, or undo, the operation you need the inverse function.

6.4 Use Inverse Functions

- Properties of Inverses
  - x and y values are switched
  - graph is reflected over the line y = x



  - You can use the Horizontal Line test to determine if the inverse of a function is also a function.
    - If a horizontal line can touch a graph more than once, then the inverse is not a function.

6.4 Use Inverse Functions

- Definition of inverses
  - Two functions are inverses if and only if
    - $f(g(x)) = x$ and $g(f(x)) = x$
  - Remember a function is when one x value only goes to one y value (lines are functions, quadratics are functions, square roots are not functions)

6.4 Use Inverse Functions

- Verify that f(x) = 6 – 2x and g(x) = $\frac{6-x}{2}$ are inverses.

ANS: [f ° g](x) = 6 – 2((6 - x)/2) = 6 – 2(3 – x/2) = 6 – 6 + x = x
[g ° f](x) = (6 – (6 – 2x))/2 = (6 – 6 + 2x)/2 = 2x/2 = x
yes they are inverses

6.4 Use Inverse Functions

- Finding inverses
  - Inverses switch the x and y coordinates
  - Switch x and y and solve for y.

- $y = 2x + 7$

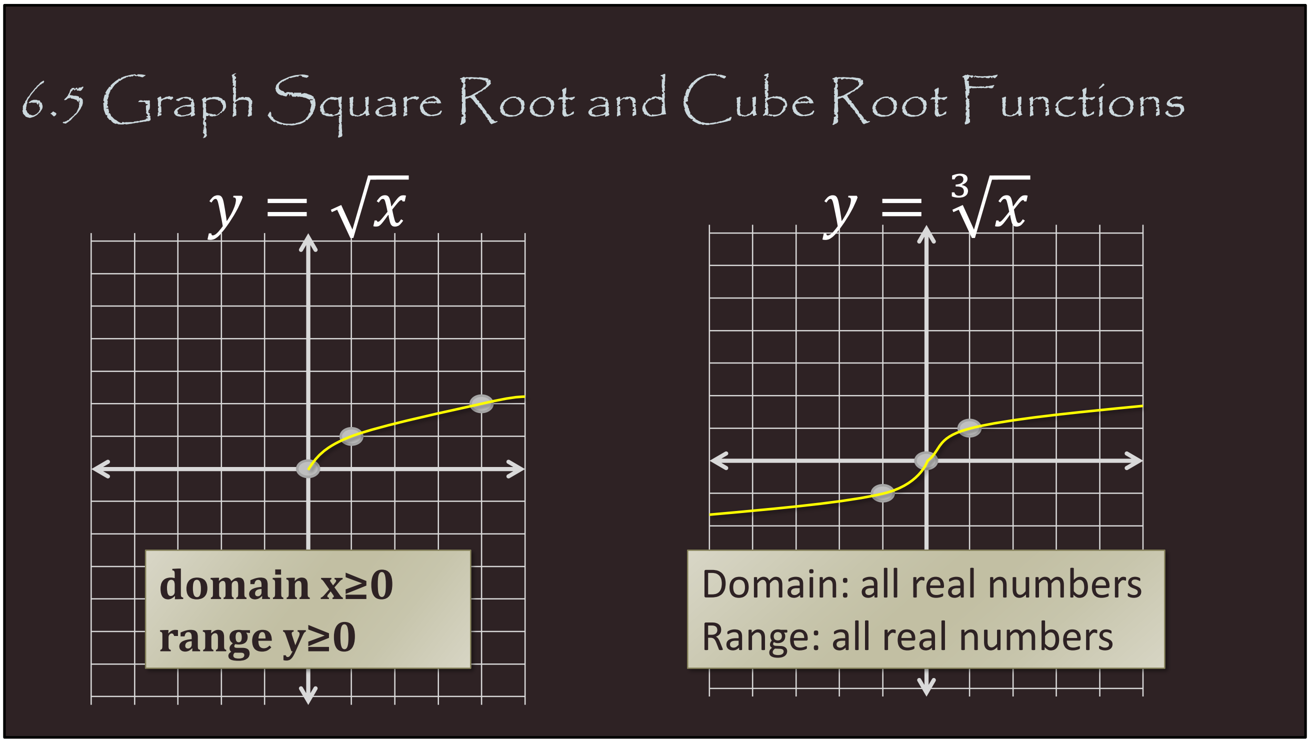
$$y = 2x + 7$$
$$x = 2y + 7$$
$$x - 7 = 2y$$
$$\frac{x - 7}{2} = y$$

6.4 Use Inverse Functions

- Finding inverses
- If the function is written as an expression
  - $f(x) = x^4 + 2, x \leq 0$
  - Rewrite f(x) as y $\quad y = x^4 + 2, x \leq 0$
    - (if problem is y=___, skip 1st and last step)
  - Switch the x and y $\quad x = y^4 + 2, y \leq 0$
  - Solve for y $\quad x - 2 = y^4, y \leq 0$
  - $y = \pm\sqrt[4]{x - 2}, y \leq 0$
  - Rewrite y as $f^{-1}(x)$ $\quad f^{-1}(x) = -\sqrt[4]{x - 2}$

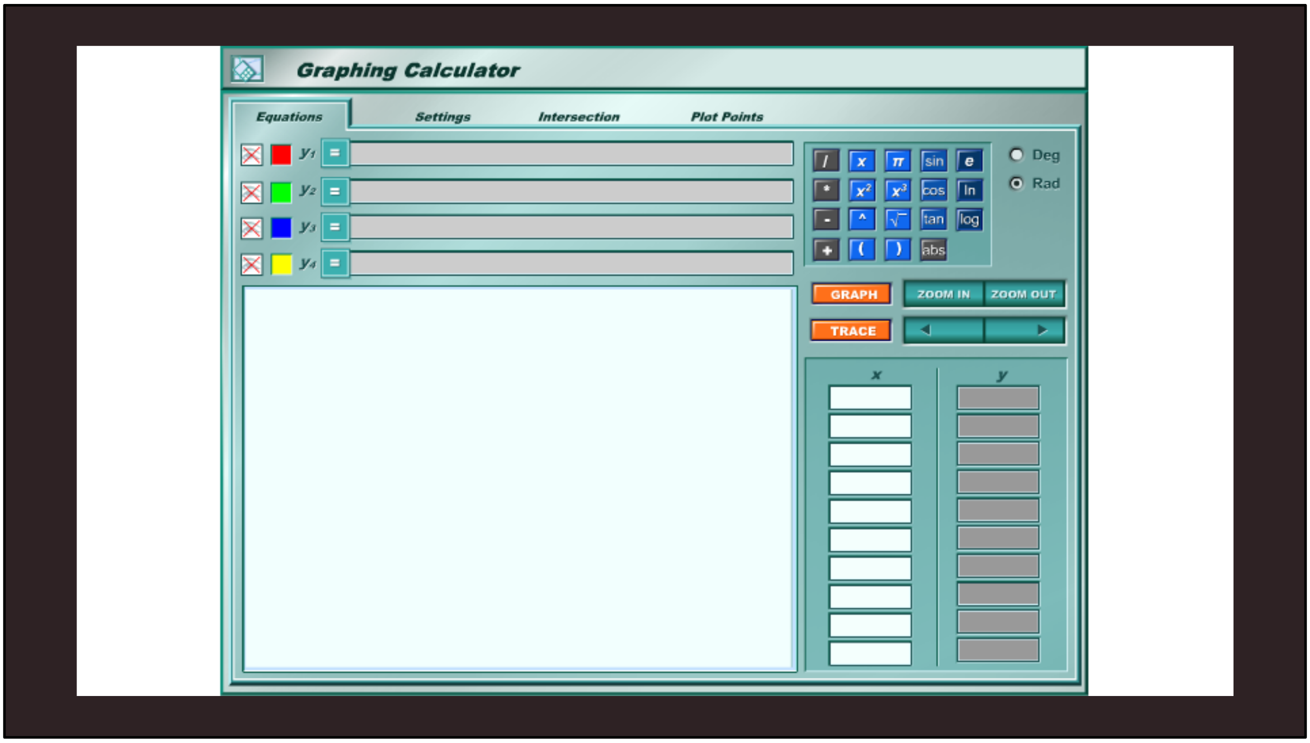
Quiz

- 6.4 Homework Quiz



$\sqrt{x}$ → domain $x \geq 0$; range $y \geq 0$

$\sqrt[3]{x}$ → domain x all real numbers; range y all real numbers



Show graphs of

$y=\sqrt{x}$

$y=2\sqrt{x}$

$y=3\sqrt{x}$

$y=1/2\sqrt{x}$

$y=-2\sqrt{x}$

$y=-\sqrt{x}$

6.5 Graph Square Root and Cube Root Functions

- How graphs transform
  - $y = a\sqrt{x - h} + k$
  - $y = a\sqrt[3]{x - h} + k$

Graphing shortcut

1. Start with the points from $y = \sqrt{x}$ or $y = \sqrt[3]{x}$
2. Multiply the y-coordinates by a
3. Move over h to the right and up k

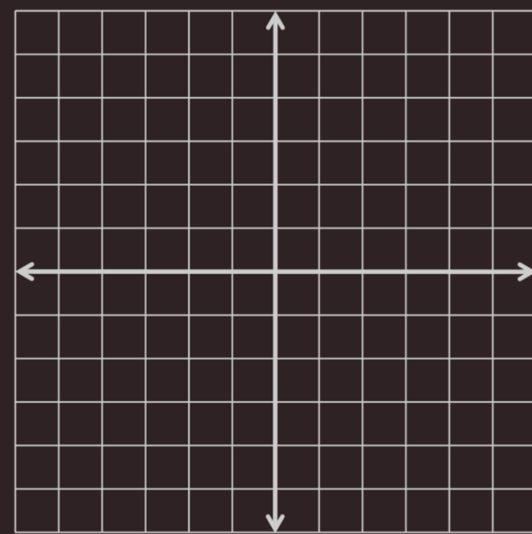
6.5 Graph Square Root and Cube Root Functions

- How did the following graphs transform?
  - $y = \sqrt{x - 3} + 4$
  - $y = \sqrt[3]{x + 3} - 5$

3 right and 4 up
3 left and 5 down

6.5 Graph Square Root and Cube Root Functions

- Graph
  - $y = -2\sqrt[3]{x-3} + 1$



6.5 Graph Square Root and Cube Root Functions

- Find the domain and range
  - $y = 2\sqrt{x+4} - 1$
  - $y = 3\sqrt[3]{x-5} + 6$

Moved 4 left and 1 down so → Domain: $x \geq 4$; Range: $y \geq -1$
Cube roots always have Domain: all real numbers and Range: all real numbers

Quiz

- 6.5 Homework Quiz

6.6 Solve Radical Equations

- Radical Equation
  - Equation containing a radical

- Steps

1. Isolate the radical
2. Raise both sides to whatever the index is (or the reciprocal of the exponent)
3. Solve
4. Check your answers!!!

6.6 Solve Radical Equations

- $5 - \sqrt[4]{x} = 0$
- $3x^{\frac{4}{3}} = 243$

$5 = \sqrt[4]{x} \rightarrow 5^4 = x \rightarrow x = 625$

$x^{4/3} = 81 \rightarrow x = +\text{-}81^{3/4} = +\text{-}27$

6.6 Solve Radical Equations

- $\sqrt{2x+8} - 4 = 6$
- $\sqrt{4x+28} - 3\sqrt{2x} = 0$

√(2x+8) = 10 → 2x+8 = 100 → 2x = 92 → x = 46

√(4x+28) = 3√(2x) → 4x+28 = 9(2x) → 4x+28 = 18x → 28 = 14x → x = 2

6.6 Solve Radical Equations

- $x + 2 = \sqrt{2x + 28}$

- Check!

$(x+2)^2 = 2x+28 \rightarrow x^2+4x+4 = 2x+28 \rightarrow x^2 +2x -24 = 0 \rightarrow (x+6)(x-4) = 0 \rightarrow x = -6, 4$

Check

-6: $-6+2 = \sqrt{(2(-6)+28)} \rightarrow -4=\sqrt{(-12+28)} \rightarrow -4 = 4$ False

4: $4+2 = \sqrt{(2(4)+28)} \rightarrow 6 = \sqrt{36} \rightarrow 6=6$ True

Quiz

- 6.6 Homework Quiz